Thickening Theories—an Analysis

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There is as yet no theoretically sound and empirically proven procedure that is fully reliable for the design of thickeners. From a logical standpoint, a sound procedure is one correctly deduced from valid premises or assumptions. As noted in Fitch's earlier work (1979), there is no reason to believe that existing mathematical models are based on uniformly valid assumptions. There is also much reason to believe that they are not. Therefore existing mathematical models or "theories" of thickening are analyzed for mathematical consistency, scientific credibility, and relevance to chemical engineering. Domains in which the models are useful approximations of reality and thus are relevant to chemical engineering practice are identified. Emendations or alternatives are suggested to bring the models more nearly into conformity with empirical results. Design procedures are suggested that, although involving assumptions that cannot be uniformly valid, would seem more reasonable than existing ones.

Introduction

Existing mathematical models for thickening are not rigorously valid, even in domains for which they are generally accepted to be so. Deficiencies in the models are not derived from mathematical inaccuracy. All models are mathematically without flaw. Each is derived "with mathematical certainty" from the conclusions implicit in the assumptions made. The deficiencies are derived rather from assumptions not valid in the real world of thickening. In certain cases the mathematical derivations may not describe what actually is occurring due to inaccurate assumptions which do not accurately represent the physical phenomena. They therefore may not have much relevance to chemical engineering practice.

Kynch Theory

The Kynch theory is the cornerstone for current mathematical models and is used in most design procedures. We examine the presuppositions of the Kynch theory, because our research concludes that the problem with existing mathematical theories of thickening occur from errors in the assumptions from which they are derived.

The Kynch theory is based on two explicit assumptions:

- One-dimensional continuity.
- Settling rate is a function of concentration only.

The assumption that settling rate is a function of concentration only is satisfied when the thickening of flocculent suspensions occurs. In the regime which has been called "zone settling" solids settle with a clear line of demarcation between supernatant suspension above and free-settling suspension below. In this regime it exhibits what has been called "line settling". The presumption is that the particles are trapped in a floc structure, that is strong enough to prevent differential settling, but is too tenuous to exhibit significant resistance to crushing (Coe and Clevenger, 1916; Fitch, 1966, 1975, 1979).

In purely theoretical studies it has often been believed that a suspension of identical noncohering spheres would also satisfy the second assumption. It is known that all the spheres would not have the same settling rate due to the formation of doublets and clusters. This does not by itself rule out the possibility that *average* settling rate, and hence also the solids flux, might not be a function of concentration. On the other hand it may follow that a suspension of identical spheres is not a valid model for what is known in industrial practice as thickening. In any case, a suspension of identical spheres is purely hypothetical. In over half a century of practice, I have never encountered one.

The Kynch theory can under some boundary conditions be shown to lead to contradictory conclusions. Therefore at least one of its premises must not be uniformly valid. Later emendments to Kynch theory, and alternatives to it, will be considered.

Figure 1 shows a Kynch plot of free-settling flux S vs. concentration c. The Kynch theory predicts that in a batch test at an initial concentration such as c_a in the plot, there will be an

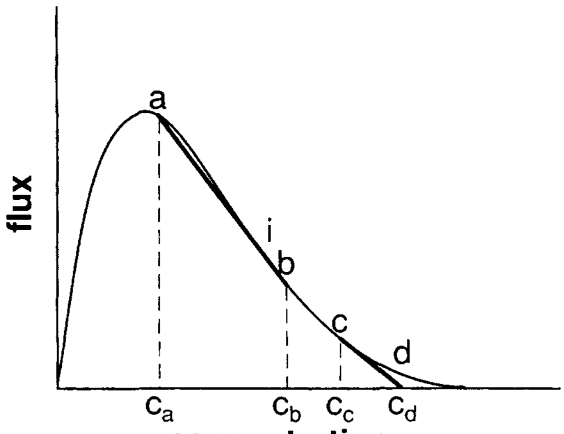

Figure 1. Kynch constructions for Coe and Clevenger type I system.

upper discontinuity from c_a to c_b, followed by a fan of Kynch characteristics from c_b to c_c. However, if the initial concentration is above that of the inflection point (i) of the Kynch curve, there will be no such upper discontinuity, (Kynch, 1953; Tory, 1961; Fitch, 1990). There will also be a compression discontinuity where the slowest Kynch characteristic at c_c is overrun by the top of the compacting zone at c_d.

D-fronts

A true or mathematical discontinuity cannot exist in a particulate system. In such a system, concentration is not well defined at a point, but as particles settle towards a region of higher concentration, their approach causes the average volumetric concentration in their local neighborhood to become greater. What is accepted mathematically in the Kynch theory as a discontinuity must in fact comprise a concentration front or boundary, with a minimum thickness comparable to the average interparticle distance s. We will call such a front a "D-front". To be represented even approximately by a discontinuity, it would have to be a concentration shock or steep front of constant shape.

In a mathematical discontinuity intermediate concentrations between that above and that below do not exist. In a D-front they do, and every concentration in the D-front must be propagating at the same velocity to maintain a constant shape. The propagation rate of a locus of constant concentration (Kynch characteristic) from the Kynch theory:

$$v_K = dS/dc$$

The slope of S vs. c would thus have to be constant through a D-front. To satisfy the material balance implicit in the one-dimensional continuity assumption, S would have to plot along the chords (a), (b), (c) and (d) in Figure 1. To satisfy the force balance implicit in the second assumption it would also have to plot along the curve, which is impossible (Fitch, 1979). The two assumptions are thus incompatible in a particulate system, so it follows from logic that at least one of them must not be rigorously valid. In order to satisfy both material and momentum balances, settling rate in a particulate system would have to be less than the free-settling rate as given by the Kynch plot. There would have to be a compression effect. However, to square with experimental results the Kynch assumptions must be a good approximation of reality under some conditions.

Force Balance

The Kynch theory assumes that settling rate is a function of concentration only. It would thus be independent of concentration gradients. In order to better understand D-fronts, settling rate would have to be lowered by a concentration gradient. We therefore abandon the assumption that settling rate is a function of concentration only and make a more complete force balance.

By one-dimensional force balance around a lamina of unit area in a suspension the following equation can be obtained, (that is, Fitch, 1979):

$$(g - Du/Dt)c\rho_s dx + (g - Du_f/Dt)(1-c)\rho_f dx = (\delta p/\delta x)dx + (\delta P_s/\delta x)dx \quad (2)$$

Since $\delta p/\delta x - g_f = u\mu/K$, Eq. 2 can be rewritten:

$$g(\rho_s - \rho_f)c - u\mu/K = [c_s(\delta u/\delta t + u\delta u/\delta x) + (1-c)\rho_f(\delta u_f/\delta t + u_f \delta u_f/\delta x)] + \delta P_s/\delta x \quad (3)$$

Mathematical Models

Existing mathematical models for thickening differ in which the terms in Eq. 3 are neglected:

The Kynch model assumes that neither the inertial terms (bracketed in Eq. 3), nor the pressure term, will have a significant effect on settling behavior. If such terms are neglected, Eq. 3 reduces to:

$$g(\rho_s - \rho_f)c = u\mu/K \quad (4)$$

The conventional assumption is that flow through the porous medium represented by the solids swarm is Darcian and that $K = K(c)$, (Michaels and Bolger, 1962; Fitch, 1966; Shirato et al., 1970). This assumption is not necessarily or uniformly valid (Kos, 1978). But assuming $K = K(c)$, it follows from the force balance that:

$$u = u(c) \quad (5)$$

Thus Kynch's tacit force balance is consistent with his kinetic assumptions. However, as argued above, his assumptions cannot be rigorously and uniformly valid in a particulate suspension. To permit the formation of D-front approximations of the Kynch discontinuities, there has to be some force which slows particles down as they approach higher concentrations. There has to be a compressive force of some kind.

Concentration Gradients

At this point we consider the effects of a concentration gradient.

Equation 3 can be rewritten as:

$$(\mu/K)u = g(\rho_s - \rho_f)c - F_a - F_c \quad (4)$$

where F_a equals the inertial terms of Eq. 3,

$$F_a = [c\rho_s(\delta u/\delta t + u\delta u/\delta x) + (1-c)\rho_f(\delta u_f/\delta t + u_f\delta u_f/\delta x)]$$

and F_c equals the compressive terms,

$$F_c = \delta P_s/\delta x$$

In a positive concentration gradient velocity u is decreasing so inertial terms augment u. The solids pressure gradient is positive, so the pressure term reduces u. The two directly counteract. System behavior therefore depends on their resultant. If inertial terms are larger than the pressure one, there is a net augmentation of settling rate, and an inertial model (to be discussed later), would be applicable with an appropriately reduced net or effective inertial term. If the pressure term dominates, particles would be retarded as they move towards regions of higher concentration.

Four kinds of solids compression can be identified: Elastic compression, static compression, osmotic compression, and dynamic compression.

Elastic compression. It would arise from random motion and collisions of particles. If assumed, it can be accounted for by including a diffusion term such as $D(\delta c/\delta x)$ in the force equation. (And if such a term is included without restriction in the equation, it bestows the property of elasticity upon the model.) Such a force, if it existed, would provide the necessary retarding force in a free-settling regime. However, for reasons to be discussed this effect seems unlikely to be significant in sedimentation.

Static compression. This arises when the particles are in contact with one another, forming a structure that has resistance to crushing. It differs from elastic compression by not being reversible. It acts only while the structure is being crushed and stores no potential energy for subsequent release to the system. Only a trace of static compression would be needed to offset the minuscule inertial forces. So it is conceivable that what have been considered as free-settling regimes actually have some trace of static compression. This could permit something very closely approximating Kynch behavior.

Osmotic compression. Auzerais et al. (1988) in a comprehensive recapitulation of flux theories, go on to develop a mathematical model attributing compressive force to osmotic pressure. This assumes that a suspension can be treated as a solution, with solids particles acting like solute molecules. In free settling, however, osmotic pressure would tend more to disrupt floccules than to retard settling rate. It would be exerted against the cohesive force holding the individual floccules together. It would not retard settling until the floccules were deformed to eliminate the macrovoids (Roberts, 1949), and the boundaries between floccules were obliterated. This presumably requires static compression. Also in many naturally flocculent or chemically flocculated suspensions the colloidal solids constitute only a very small fraction of the total weight present. The contribution of the remaining large particles to osmotic pressure might be very small indeed. Therefore it is at least questionable that the axiom assumptions of the Auzerais et al. mathematical model are generally realistic.

Dynamic compression. It is well known in theoretical analysis that settling particles do slow down as they approach a discontinuity. Dixon (1977) characterizes the force causing this retardation as "dynamic compression." It arises from the excess local pressure needed to squeeze fluid out from between approaching particles. It is manifested in a record changer as the force which prevents the dropping record from crashing onto the one below.

In the concentration gradients occurring in the real world inertial forces, together with dynamic and osmotic pressure ones, however small, always will be present. Static pressure may be present. As noted above, where their resultant is negative, the inertial model is applicable. Where it is positive, the suspension is mathematically "in compression" and the conventional approaches for determining concentrations through a steady-state compression zone are applicable. (Fitch, 1966; Adjordan, 1975; Chandler, 1976; Kos, 1978; Tiller and Chen, 1988; and Landman, White, and Buscall, 1988).

In the absence of concentration gradients, settling rate becomes u^*, and from Eq. 4:

$$(\mu/K)u^* = g(\rho_s - \rho_f)c \tag{5}$$

Eliminating μ/K between Eqs. 4 and 5:

$$u = u^* - F_R u^*/[gc(\rho_s - \rho_f)] \tag{6}$$

where F_R is the resultant of F_a and F_c or in flux variables:

$$cu = cu^* - F_R/[g(\rho_s - \rho_f)]$$

$$G = S[1 - F_R/(g\Delta\rho)] \tag{7}$$

Note that Eqs. 6 and 7 are simply shorthand formulations of force balance Eq. 3, with operationally meaningful variables.

Compression Fronts

The shape of a compression region, be it a zone or a front, will be determined by the functionality of the F_R term. Its thickness will in general depend upon how rapidly P_s varies with concentration. Easily compressed sediments will result in thin fronts.

Existing compression models all assume that inertial forces are negligible, and that both Darcian permeability K and solids pressure P_s are functions of concentration alone. The latter should be valid for static and osmotic compression and would be true of elastic compression if indeed there were a random motion of particles analogous to the thermal movement of molecules in a gas. It would not be true for dynamic compression.

A basic equation for the conventional model is (Fitch, 1990):

$$\delta c/\delta x = (1 - u/u^*)(dc/dx)^* \tag{8}$$

and for steady-state fronts (Fitch, 1966):

$$dx/dc - [S/(S-G)(dx/dc)^* \tag{9}$$

where

- $u^* = g(\rho_s - \rho_f)Kc/\mu$ (From Eq. 4). It is the settling rate in the absence of solids pressure gradients, and thus is the settling rate which would be observed experimentally in a free-settling regime.
- $(dc/dx)^* = g(\rho_s - \rho_f)c/(dP_s/dc)^*$. It is the concentration gradient that would be observed in a batch settling test after all subsidence is complete and hence u is zero.

$S = cu^*$, and is the Kynch settling flux.

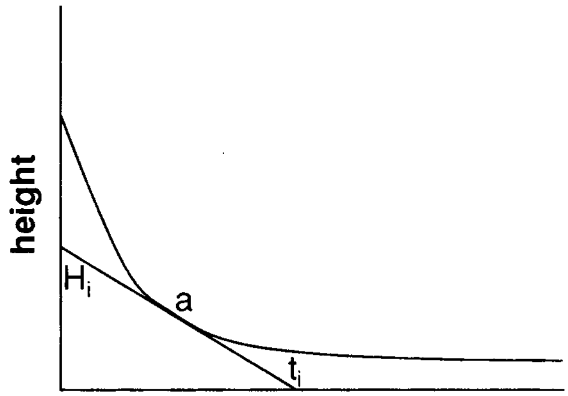

Figure 2. Kynch constructions for $1/c_a$ and $1/S_a$ on settling plot.

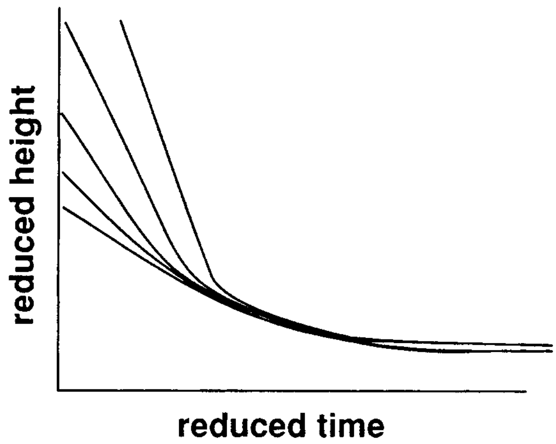

Figure 3. Reduced settling plots for a series of runs.

Other forms of Eqs. 8 and 9 have been derived *de novo* in the literature. However, they all start from the same premises and solve the same problem, namely the concentration gradient through a compression region. They are interderivable by substitution of variables. They do not represent different models of system behavior, but only different ways of expressing it. Equations 8 and 9 have the practical advantage of being directly related to Kynch theory, and make certain relationships obvious.

The term $(S/S-G)$ can be interpreted as a "stretch factor", showing how much greater dx/dc would be in a steady-state compression zone than it would be in a batch test after all subsidence was complete. At points (a) and (b) in Figure 1, S becomes equal to G and the stretch factor becomes infinite. Therefore under the conventional assumptions a steady-state front from c_a to c_b would have to be infinitely deep and would take forever to develop. The same would be true for the front from c_c to c_d. They could not develop in a batch test of finite initial depth. Thus the actual fronts developing where the Kynch theory would predict discontinuities, with intuition suggesting concentration shocks, would have to be unsteady-state ones whose shape was changing with time.

Batch Settling Data

At this point we do the scientific thing and check experimentally whether the fronts from (a) to (b) and (c) to (d) in Figure 1 are in fact shocks, or are expanding fronts.

A batch settling test gives directly a plot of interface height H vs. time t. A tangent to the settling plot at some point (a), (Figure 2) has a slope dH/dt that is equal and of opposite sign to the subsidence rate R of particles just below the suspension-supernatant interface at that time. It thus gives the settling rate at point (a) under Kynch assumptions. This tangent intersects the H axis at H_i and the t axis at t_i. From Kynch theory:

$$H_i = c_0 H_0 / c_a$$

$$t_i = c_0 H_0 / S_a$$

Therefore, if both scales are divided by c_0H_0 the tangent intercepts will fall at $1/c_a$ and $1/S_a$ respectively on the new axis scales, over the range for which Kynch assumptions are valid. The relationship between S and c is independent of initial height and time. Therefore, all reduced settling plots made on the same suspension should coincide over the Kynch range regardless of the initial heights and concentrations. This is demonstrated in Figure 3, which shows reduced settling plots for batch settling tests numbers 24 to 31 from Tory's thesis (Tory, 1961). As discussed elsewhere (Fitch, 1990), these are the best experimental data available for analysis. Data for the runs are shown in Table 1.

Table 1. Reduced Settling Plots for Batch Settling Tests

Run No.	Init. Conc. kg/m³	Init. Hgt. cm
24	28.59	48.25
25	24.00	57.60
26	18.99	72.90
27	13.86	100.00
28	9.94	140.00
29	19.04	147.35
30	24.01	116.50
31	28.68	97.40

Although it turns out that settling rate at any given concentration is somewhat dependent upon initial concentration, all plots do come close to coinciding and have essentially the same shape through the Kynch range.

The significant changes in a batch settling curve are difficult to see and interpret directly. They are much more evident in a plot of subsidence rate vs. height. Such plots were introduced by Tory in his thesis (1961), and they are here called "Tory plots".

Figure 4 shows a Tory plot corresponding to Figure 1, as would be obtained by Kynch theory. The suspension-supernatant interface would subside at its initial concentration and at constant rate until it reached point (a). At this point it encounters the fastest rising Kynch characteristic at (b), and there is a discontinuity in settling rate. From (b) to (c) the interface encounters Kynch characteristics of ever increasing concentration and lower settling rates. At point (c) the Kynch characteristic is overrun by the rising compression level, and there is a discontinuity from (c) to (d).

Figure 5 shows experimental Tory plots of subsidence rate vs. reduced height for the same tests shown in Figure 3. The

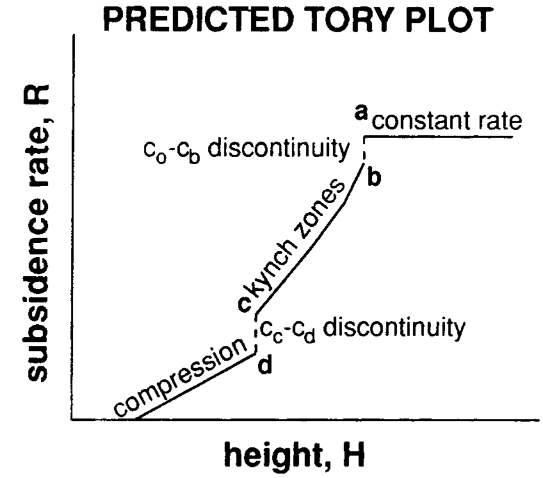

Figure 4. Tory plot as predicted by the Kynch theory.

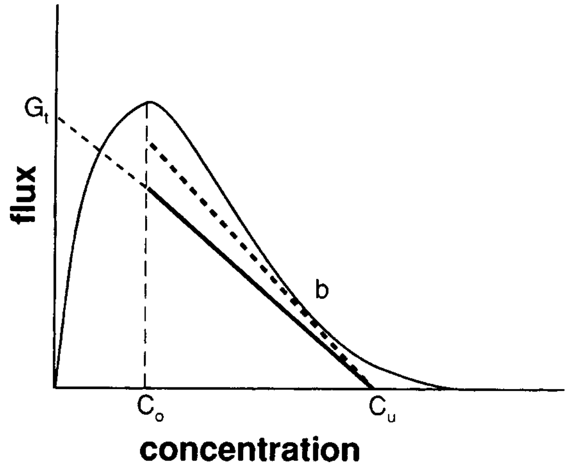

Figure 6. Yoshioka construction.

plots were calculated and drawn from the Tory data by computer program SEDANAL, to be described later.

Tory plots magnify any deviations in local slope, including those due to experimental error or noise. However, they show a section from initial height to some point such as (a) on the uppermost plot that is on the average constant. The suspension-supernatant interface subsides at its initial concentration until at point (a) the interface encounters the top of a concentration front propagating up from below. If the initial concentration is above c_i it will be a Kynch front. If it is lower, it will be a front corresponding to the discontinuity of the Kynch theory. At point (b) the interface encounters the highest of the Kynch characteristics. Thus from (a) to (b) the interface is passing through a D-front. The D-front region has different shapes in the different plots on Figure 5, so apparently the fronts do not have a constant shape. The plots in Figure 5, however, show reduced heights rather than actual interface heights. It is not immediately evident that the shapes would not appear constant in plots of unreduced height. The following comparisons show directly that the D-front is not of constant shape, but is expanding linearly with time.

In Figure 5 there are three pairs of tests in which the initial concentrations c_0 are the same, but initial heights H_0 are different. They are runs 26 and 29, 25 and 30, and 24 and 31. Those at the same initial concentrations coincide not only through the Kynch region, but through the upper D-front region as well. Since they are plotted in reduced height coordinates and coincide in these coordinates, they would not do so in unreduced ones. The D-front is expanding linearly with time just like the fronts created by Kynch characteristics. Apparently linear characteristics are propagating upward through the D-front at rates different than predicted by Kynch theory as well as at rates that are some function of initial concentration.

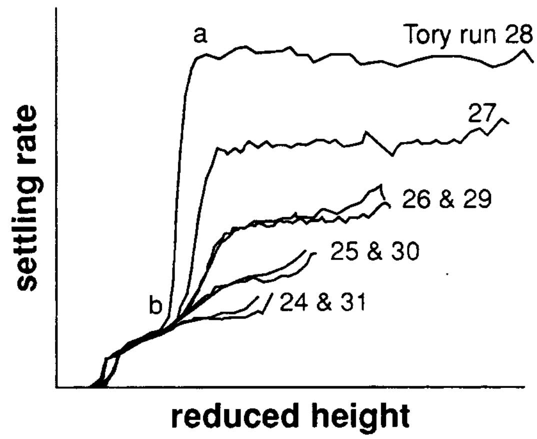

Figure 5. Experimental Tory plots for runs shown in Figure 3.

The evidence is not so obvious in the case of the lower or compression discontinuity. Plots coincide through this range only for tests made with the same value of c_0H_0, or amount of solids per unit area. However, the region below the Kynch regimes contains two ranges (Fitch, 1990). The lower or high concentration range of the Tory plot is linear, corresponding to Robert's empirical model for compression (Roberts, 1949). Between the Roberts domain and that of Kynch there is a transition region or front. Such a front is observed generally in settling of flocculent suspensions. By Occam's razor (William of Occam, 1285–1349) we would expect it to have much the same characteristics as the upper D-front.

Thus the concentration shocks expected intuitively in batch settling where Kynch theory would predict discontinuities do not form. They are replaced by D-fronts expanding linearly with time.

The problem would not arise in one-dimensional steady-state thickening. Figure 6 shows a Yoshioka construction (Yoshioka et al., 1957) for continuous thickening. The operating or material balance line is below the Kynch curve at all points between c_a and c_d. The stretch factor remains finite and also the thickness of the D-front. However, if the thickener solids flux G_t were increased, the stretch factor would increase at all points, and the front would expand. When the critical flux is approached, the operating line would become tangent to the Kynch curve at point (b). The stretch factor would become infinite at that point, and a critical zone of concentration c_b would fill the thickener. This is consistent with the finding that an ever increasing front would form in batch sedimentation. However, this is of more theoretical than practical interest, because as shown in Fitch (Oct. 1990), the free-settling domain in operating thickeners is not one-dimensional.

Kynch Region

In the region from c_b to c_c (Figure 1) Kynch characteristics are propagating to the surface. Except at the very origin, the concentration gradients developed are low, and compression forces are very small with respect to gravitational ones. Therefore we would expect the charactertistics to propagate very much as predicted by Kynch theory. However, starting at time zero, a D-front should develop just above the bottom of the column. This creates the initial gradient out of which Kynch characteristics propagate.

At this point it has been shown that the Kynch theory is valid to a high degree of approximation over the range in which characteristics are propagating to the interface. It is not valid in the region in which it would predict discontinuities. The addition of solids pressure terms which are small with respect to gravitational ones, but are larger than inertial ones, results in predictions generally consistent with experimental results. We next consider the case where inertial terms dominate compression ones.

Inertial Fronts

In a compression front the inertial forces resulting from deceleration of the particles are absorbed by solids compression. Solids are slowed down at all points until at the bottom of the front their actual settling rate becomes equal to the free-settling rate at that point. They thus join the suspension below the front at the same concentration already existing there and add on to them. This causes upward propagation of the D-front.

In an inertial front the inertial forces are thus not absorbed. To satisfy material balance solids in positive concentration gradient must be settling at greater than their free-settling rate.

Two mathematical models have been given in the literature for inertial fronts: That of Dixon, and the fluidization model of Jackson (1963), and of Pickford and Baron (1965).

It was shown by Dixon et al. (1976) that a special case solution for an inertial shock is mathematically possible. It assumes one-dimensional continuity and that compressive forces are negligible. It can be explained as follows:

In Figure 1 the Kynch theory would predict continuity characteristics from point (b) to point (c). Figure 7 shows the analogous construction for an inertial shock from point (c) to point (d). The material balance line runs everywhere above the Kynch curve through this range. Unless the initial concentration is below c_c it will not run below the Kynch curve at any point. Inertial terms will depend on the actual velocity gradient, and hence settling rates would be augmented in a positive concentration gradient. Under the assumptions of the Dixon model the concentration gradient would adjust in such a way that the actual settling fluxes would plot on the material balance line. Material and momentum balances would thus both be satisfied. However, this explanation would not be generally valid. It does not explain what would happen in the real world under boundary conditions such that the material balance line runs below the Kynch free-settling plot, as in Figures 1 and 6.

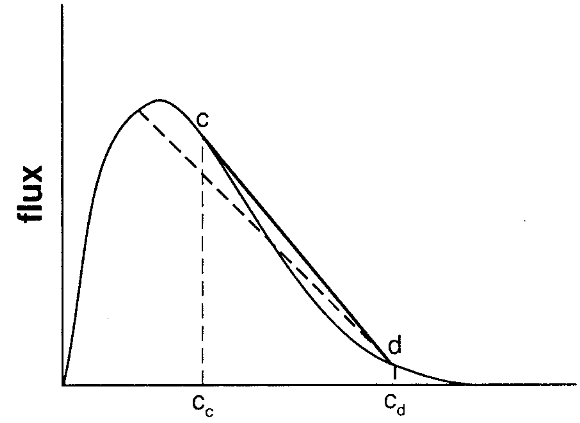

Figure 7. Construction for hypothetical Dixon shock.

As a matter of empirical fact, suspensions of larger particles do collapse directly from dilute suspension to compression. This has to be explained some way. Since the Dixon model does not do so, it would appear to derive from some assumption that is not generally valid.

Fluidization

Particulate fluidization may be considered a steady-state sedimentation in which the system has a transport rate equal to the superficial or phase velocity of the fluid phase. Concentration adjusts until the settling rate of the solids becomes equal and opposite to the transport rate.

In particulate fluidization the particles are usually large and have a substantial settling rate. Therefore inertial effects would be relatively large, while compressive effects are small, as compared to thickening suspensions. Thus it could be expected that the inertial or Dixon model would be relevant.

Under this model the state of particulate fluidization (or sedimentation) would be unstable. If a nodule of higher concentration formed by perturbation, the particles in it would have a settling rate lower than the general settling rate and would be dragged upward by the fluid flow. Because of the inertial terms, the settling rate at any concentration above and into the nodule would be greater than its free-settling rate. At the same concentration in the reverse concentration gradient below the nodule it would be less than the free-settling rate. The flux of particles into the nodule would therefore be greater than that out of the nodule. The nodule would thus collect into itself particles that it overran and would increase in concentration as it did so. Therefore its concentration and also its rate of growth would be increasing. The growing concentration spike would leave a region of lowered concentration beneath it. Such a doublet would be unstable gravitationally and would overturn before developing far. The turbulent appearance of such fluidized beds could be attributed to this continual upsetting.

The state of uniform fluidization is also propagationally unstable where the second derivative of S with respect to c is positive (Fitch, 1966).

It should be noted that a perturbation could not propagate upward under this model faster than the rising rate of the fluid.

JPB theory

Jackson (1963), and Pigford and Baron (1965) propose a mathematical model for fluidization, which will here be labeled the "JPB" model. It assumes initially the same force balance later used by Dixon, but reaches different conclusions. It presupposes, like Dixon, that the solids phase will remain effectively continuous and will not break up into discontinuities. In addition it presupposes and imposes a wave train solution. Then, by linearized analysis, it is deduced that an initial perturbation of any wavelength will grow, and therefore that the state of uniform fluidization is unstable. Pigford and Baron further deduce that the growth rate (for short wavelengths) would be inversely proportional to the square root of the wavelength. The fastest-growing and thus dominant wave would therefore have zero wavelength, would grow at an infinite rate and would propagate at zero velocity.

Pigford and Baron reject this as "an obviously faulty conclusion." The fault lies not in the deductions. Although it does not account for the upward displacement of the spike once it is formed, it is about as good a recipe for the formation of a concentration spike as the presuppositions of their model permit. The fault lies in the presuppositions.

As noted, the JPB model presupposes and imposes a wave train solution. However, there is no scientific mechanism apparent that would support the propagation of wave trains.

In a true or molecular gas a local density perturbation will oscillate between trough and crest because of its elasticity. Energy transfers reversibly between potential energy of compression and kinetic energy of movement. This gives rise to a train of waves propagating in all directions. There is no reason apparent to believe that production of solids pressure in a suspension will be reversible, and thus that the solid phase will be elastic. Even if the solid phase did exhibit some elasticity, any elastic waves that were generated would be damped out immediately. In a true gas, the molecules fly without resistance and without loss of energy along their free paths. In a suspension the particles would be subject to hydraulic drag all along the analogous path, which would rob them of the kinetic energy they would need to regenerate a solids pressure. Thus the fact that a wave train solution would be appropriate for a true gas is insufficient basis for assuming the same for a particle gas.

The Dixon model cannot be generally valid for inertial fronts, and there is no physical basis for the wave train JPB model. Some other model will have to be contrived to be consistent with what actually happens.

Noncontinuity Behavior

Kynch's theory for propagation of a discontinuity presupposes that settling rate is a function of concentration, and that the concentrations c_a above and c_b below the discontinuity remain the same. However, if particle settling rate were purely a function of concentration, then particles from a region of concentration c_a above a discontinuity would not change velocity until they had entered the neighborhood of different concentration below. The inconsistency of assuming that particles would slow down *before* they reached the discontinuity, collapsing onto the top of it at the same concentration that existed below (which would be necessary for the discontinuity to propagate upward) was pointed out by Hassett (1965).

Under the above assumptions, the volume of the lamina just below a discontinuity would be infinitesimal, the flux of particles into it from above would be finite, and the flux of particles out of it would be zero (since the lamina would be subsiding at the settling rate of the particles in it). So the concentration just below the discontinuity would not remain constant. It would increase instantaneously and without limit. Obviously the model loses all validity at the point the particles come into contact with one another, and this would represent a bounding condition. So under the model, we might predict that the concentration in the lamina would jump to its compression value, c_d, after which succeeding particles would be sterically constrained to pile up above it at the same concentration. A Dixon (1977) type discontinuity with free-settling suspension at c_a above and compression sediment below would be formed. Thus a plug of compression point sediment would form at the site of the original discontinuity. The top of the plug would then propagate at a rate determined by the original concentration of the suspension and the concentration in the plug. The plug would increase in thickness at a rate:

$$dh/dt = u_d - (c_a u_a - c_d u_d)/(c_d - c_a) \qquad (10)$$

Below the plug the particles would be settling at a rate u_b, which is greater than u_d, so particles below the plug would simply settle away from it leaving a void. Such a concentration inversion is gravitationally unstable, and unless constrained by some structure in the suspension would almost immediately upset.

Kynch's material balance is not valid because it does not take into account the instantaneous increase in concentration below from c_b to c_d.

In a mathematical discontinuity there is no clear distinction between particles slowing down just above or just below a hypothetical discontinuity. When inertial terms are admitted, the ambiguity disappears. Particles will not slow down until they have plunged some distance into the higher concentration region below. They would there increase the concentration until the compression point was reached.

If a concentration gradient were somehow created, with free-settling concentrations above and below, the particles above the top of the gradient would plunge below the top. The concentrations below would increase until either the plug upset, or a spike of compression pulp was created. Concentrations above the top of the gradient region would not be increased earlier, and the gradient could not spread.

The picture which emerges is that an impact boundary would be created. Settling particles from above would rain down onto the boundary like raindrops falling into a pool of water. Raindrops do not lose their momentum until they pass the surface and join the water below. Likewise the settling particles would not dissipate all their momentum until they impacted the compression pulp below.

The above line of reasoning should be of theoretical interest, and would seem to merit further analysis. It is, however, of little relevance to thickening, since inertial fronts apparently do not occur in practice. It will not be further argued here.

Short-Circuiting

Short-circuiting occurs both at the higher free-settling con-

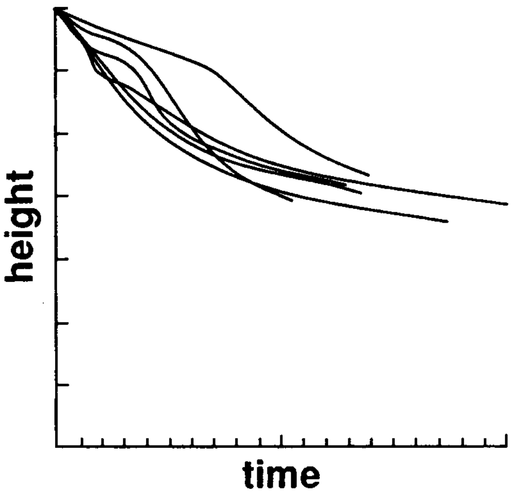

Figure 8. Repeated settling plots for concentration in the short-circuiting range.

centrations, and in the top of a compression zone. In a compression zone it is visible as channelling. In free-settling it is manifested by settling rates that increase with time (Fitch, 1990). In the short-circuiting region neither settling rate nor Darcian permeability are functions of concentration alone. In the free-settling range short-circuiting is wildly unpredictable. Figure 8 shows batch settling plots all made on exactly the same column of pulp at a concentration in the short-circuiting range. The only difference was in the type and amount of agitation the column received initially. In some runs the solids were resuspended by inverting the column several times. In others the suspension was churned with a rubber stopper on the end of a glass rod. Even in cases where attempts were made to duplicate the stirring history, settling was not the same from one run to another.

While short-circuiting behavior cannot at this time be predicted, we still have to design thickeners. Therefore the effects of this uncertainty are considered.

Short-circuiting increases permeability. If it occurs in the region where the concentration would otherwise be critical or limiting, the unit area required would be less than that predicted. Where it occurs in the compression zone it decreases the stretch factor and the compression depth will be less than that predicted. Thus neglect of the effects of short-circuiting will not result in underdesign of a thickener.

Design Procedures

There is at present no mathematical model for thickening that is theoretically sound and empirically proven. The classical Coe and Clevenger for unit area has been found to underdesign sometimes by as much as 50 percent. One reason may be that settling rate at any given concentration is a function not only of concentration, but of initial concentration as well. Another reason may be that the critical concentration occurs in the compression zone, which the Coe and Clevenger procedure does not take into consideration.

The Talmage and Fitch procedure for unit area seems usually to result in overdesign. This probably occurs because the critical point is not easily located on the settling curve. Traditional empirical methods usually place it at too high a concentration. The method is also theoretically unsound because the concentration that is critical for continuous operation does not arise as a Kynch characteristic in a batch test (Fitch, 1990).

The problems with the Talmage and Fitch method seem partially rectified in the Tory plot extrapolation method proposed in Fitch (1990). It is very easy to discern the compression point in such a plot, and settling rates for the higher Kynch characteristics that do not arise in a batch test are found by extrapolation of the Kynch range. It clearly shows the presence of short-circuiting if it is present, but does not tell exactly what to do about it.

In most cases it appears that any overdesign resulting from use of the extrapolation method will not be too great. We cannot be sure of this until a lot more experimental work has been done showing how actual thickener performance compares with that predicted. However, the extrapolation method seems the most reasonable for predicting unit area at the present stage of knowledge.

Compression depth can be predicted by integrating Eq. 9. The data needed are $(dx/dc)^*$ and S as functions of c through the compression regime. The former can be determined from the concentration profile in a batch settling test after all subsidence is complete. However, we do not have at present any easy way to determine S. First, we do not know how to handle the effects of channeling. Secondly, direct determination in a batch test of S as a function of c for the compression region requires sophisticated measuring techniques.

In the compression range very little depth is required for the first part of the compaction. Most of it is required for compressing the pulp to higher concentrations. Channeling is observed only in a band at the top of the compression zone in batch tests, and therefore only at lower concentrations. Thus the overdesign of compression depth resulting from neglect of short-circuiting effects should not normally be too great. At the present state of knowledge it would seem reasonable to neglect them at least as a first approximation.

Experimentally it would be complicated to measure S as a function of c in batch compression. A not too implausible assumption might be that in the absence of short-circuiting, S would be approximately the same function of c through compression as it is in free-settling. S as a function of c is determined in the simple extrapolation procedure for unit area cited above. If one is prepared to accept this approximation, a corresponding compression depth is easily predicted.

The corresponding design procedure would be to make a batch settling test on thickener feed suspension, preferably in a column at least one meter high and ten centimeters in diameter. Interface height readings should be taken at very frequent intervals, until the solids are all in compression. Then after all subsidence is complete, samples should be taken to determine the concentration profile. These data can then be fed into a computer and processed.

Computer Programs

I have three computer programs available on 5 1/4 in. floppy disks, formatted for PC or compatible computers, running on the DOS operating system. (These are also available from the *AIChE Journal* editorial office.) In each case the compiled program may be run directly and its source program is written in Turbo Pascal. A document file describes the program.

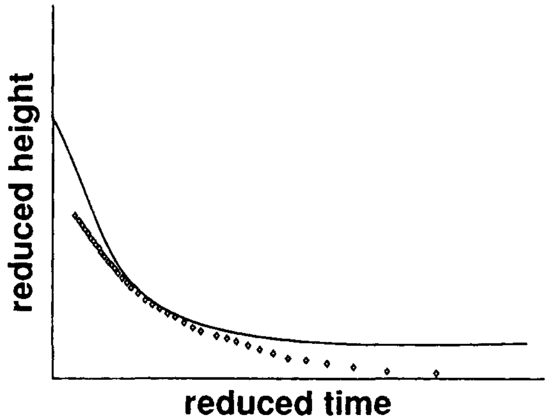

Figure 9. Settling plot with points calculated from extrapolated Kynch region equation.

Program THICKNER enters batch settling data either from a file, from the keyboard, or from the keyboard in real time. In the last case the interface height readings are entered as they are taken. The computer keeps track of and enters the elapsed time. Data can be edited and corrected. The program then calculates thickener areas required by any of four methods described in the literature, including the extrapolation method suggested above.

Program sedimentation analysis (SEDANAL) calculates Tory plots, and compares both settling plots and Tory plots for any group of runs. Graphs can be made to any scale. Most of the plots displayed in this article were drawn by SEDANAL. It also determines parameters for the empirical equation given in Fitch (1990) for the Kynch region of a settling plot. Figure 9 shows a settling plot (solid line) and points calculated from the corresponding empirical equation. The program also extrapolates the compression leg of the plot to predict the interface height at infinite time.

Program compression analysis (COMPANAL) is under development. It fits an empirical equation to the concentration profile existing in a batch test after subsidence is complete. Eventually it will predict concentration profile in the compression zone of a continuous thickener. At present there is not sufficient data against which to validate the equation. At present the equation used is:

$$X/Xzero = [(\mathrm{cinf} - c)/(\mathrm{cinf} - \mathrm{czero})]^n$$

Figure 10 shows how well it fits the data of Shin and Dick (1980). The solid line is calculated from the equation. The data points are those read as closely as possible from the average value line given in Figure 5 of the article.

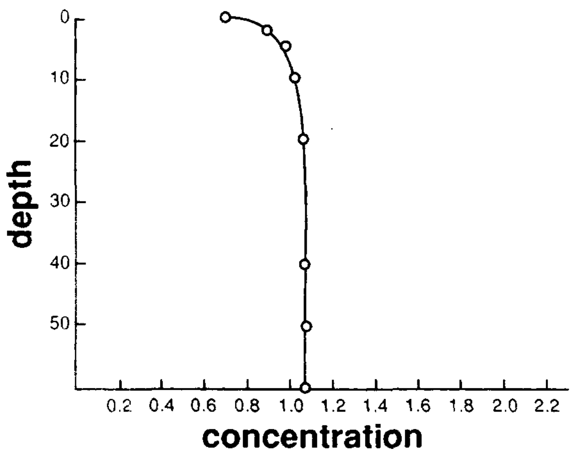

Figure 10. Empirical equation for concentration profile and data points.

Notation

c = volumetric concentration of solids
c_i = concentration at inflection point of Kynch plot
c_{inf} = concentration approached asymptotically at bottom of a compression zone
c_{zero} = concentration at top of a compression zone
F_a = forces due to acceleration
F_c = forces due to compression effects
F_R = resultant of acceleration and compression effects
g = acceleration of gravity
G = solids flux
H_i = height intercept of a Kynch tangent
K = Darcian permeability
P = fluid pressure
P_s = solids pressure
t = time
t_i = time intercept of Kynch tangent
u = settling velocity of solids
u^* = settling velocity of solids in absence of concentration gradients
u_f = velocity of fluid
v_k = propagation velocity of Kynch discontinuity
x = distance in direction of settling
X = distance from top of a compressed zone
x_{zero} = arbitrary constant

Greek letters

μ = viscosity of fluid
ρ_f = density of fluid
ρ_s = density of solids

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